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# The Meaning of Coherence in Weak Decay Processes: 'Neutrino Oscillations' Reconsidered

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**Abstract**

If neutrinos are massive, 'lepton flavour eigenstates' are absent from the amplitudes of all Standard Model processes. Measurements of $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ and of the MNS matrix are shown to exclude the production of such states in pion decay. Giunti's objection to this conclusion is shown to be invalid. It is pointed out that 'coherence' is a property not of production processes but of the path amplitudes of quantum experiments. Incoherent production of neutrinos of different mass modifies the usual oscillation phase derived assuming production of a lepton flavour eigenstate. The new prediction can be tested in long baseline experiments by comparison of '$\nu_\mu$ disappearance' of neutrinos from pion and kaon decay, or by searching for a neutrino energy dependence of the phase of atmospheric neutrino oscillations using future data from the IceCube neutrino detector.

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In the Standard Electroweak Model, the coupling of a charged lepton, $\ell_i$, of generation $i$ and a massive neutrino, $\nu_j$, of generation $j$ to the W-boson is described by the leptonic charged current:

$$J_\mu(CC)^{lept} = \sum_{i,j} \overline{\psi}_{\ell_i}\gamma_\mu(1-\gamma_5)U_{ij}\psi_{\nu_j} \tag{1}$$

where $U_{ij}$ is the Maki-Nakagawa-Sakata (MNS) [1] charged lepton flavour/ neutrino mass mixing matrix. Table 1 shows the elements of this matrix as estimated [2] from experimental measurements of atmospheric and solar neutrino oscillations. The non-diagonal nature of this matrix gives evidence for strong violation of generation number (or lepton flavour) by $J_\mu(CC)^{lept}$. Conservation of generation number corresponds to a diagonal MNS matrix with $\nu_1 = \nu_e$, $\nu_2 = \nu_\mu$ and $\nu_3 = \nu_\tau$. This is the conventional massless neutrino scenario. With massive neutrinos and a non-diagonal MNS matrix, '$\nu_e$', '$\nu_\mu$' and '$\nu_\tau$' do not exist as physical states. That is, they do not appear in the amplitude for any physical process. This is because the neutrino charged lepton couplings to the W are completely specified by $J_\mu(CC)^{lept}$ which contains only the wavefunctions of the neutrino mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$.

Indeed, the introduction of such 'lepton flavour eigenstates' as a linear superposition of neutrino mass eigenstates leads to predictions that are excluded by experiment. To show this, the pion decays: $\pi \to e\nu$ and $\pi \to \mu\nu$ may be considered. The invariant amplitude to produce the anti-neutrino mass eigenstate, $\overline{\nu}_i$, in association with a charged lepton $\ell^-$ ($e^-$ or $\mu^-$) is:

$$\mathcal{M}_{\ell i} = \frac{G}{\sqrt{2}} f_\pi m_\pi V_{ud} \overline{\psi}_\ell (1-\gamma_5) U_{\ell i} \psi_{\overline{\nu}_i} \tag{2}$$

where $G$ is the Fermi constant, $f_\pi$ and $m_\pi$ are the pion decay constant and mass respectively, and $V_{ud}$ is an element of the CKM [3] quark flavour/mass mixing matrix. Introducing a 'lepton flavour eigenstate', $\psi_{\overline{\nu}_\ell}$, as a coherent superposition of the mass eigenstates $\psi_{\overline{\nu}_i}$:

$$\psi_{\overline{\nu}_\ell} \equiv U_{\ell 1}\psi_{\overline{\nu}_1} + U_{\ell 2}\psi_{\overline{\nu}_2} + U_{\ell 3}\psi_{\overline{\nu}_3} \tag{3}$$

gives, for the invariant amplitude of the decay process $\pi^- \to \ell^- \overline{\nu}_\ell$:

$$\mathcal{M}_\ell = \mathcal{M}^D_{\ell 1} U_{\ell 1} + \mathcal{M}^D_{\ell 2} U_{\ell 2} + \mathcal{M}^D_{\ell 2} U_{\ell 3} \tag{4}$$

where $\mathcal{M}^D_{\ell i}$ ($D$ is for 'diagonal') is given by Eqn(2) with the replacement $U_{\ell i} = 1$. If all neutrino masses are much smaller than the electron mass, it follows that:

$$\mathcal{M}^D_{\ell 1} \simeq \mathcal{M}^D_{\ell 2} \simeq \mathcal{M}^D_{\ell 3} \simeq \mathcal{M}^D_{\ell 0} \tag{5}$$

where $\mathcal{M}^D_{\ell 0}$ is the invariant amplitude calculated on the assumption of massless neutrinos. With two flavour ($e$-$\mu$) mixing so that:

$$\begin{pmatrix} U_{e1} & U_{e2} \\ U_{\mu 1} & U_{\mu 2} \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \tag{6}$$

and $U_{e3} = U_{\mu 3} = 0$, Eqns(2)-(6) lead to the prediction:

$$R_{e/\mu} \equiv \frac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left[\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right]^2 \left(\frac{\cos\theta_{12} + \sin\theta_{12}}{\cos\theta_{12} - \sin\theta_{12}}\right)^2 \tag{7}$$

Allowing for radiative corrections [4, 5] the world average experimental value $R_{e/\mu} = (1.230 \pm 0.004) \times 10^{-4}$ [6] leads to a determination of the mixing angle $\theta_{12}$ via the relation:

$$\left(\frac{1+\tan\theta_{12}}{1-\tan\theta_{12}}\right)^2 = 0.9976 \pm 0.0032 \tag{8}$$

The result is:

$$\tan^2\theta_{12} = (3.6^{+16.0}_{-3.2}) \times 10^{-7}$$

or

$$\tan^2\theta_{12} < 3.0 \times 10^{-6} \text{at } 95\% \text{ CL}$$

These results are evidently incompatible with the two-flavour mixing results obtained from the study of solar neutrino oscillations [2] where $\theta_\odot \simeq \theta_{12}$:

$$0.22 < \tan^2\theta_\odot < 0.71 \quad \text{(LMA solution)}$$

$$0.47 < \tan^2\theta_\odot < 1.1 \quad \text{(LOW solution)}$$

| $j$ | 1 ($\nu_1$) | 2 ($\nu_2$) | 3 ($\nu_3$) |
|---|---|---|---|
| $i$ | | | |
| 1 ($e$) | $0.79 \pm 0.12$ | $0.57 \pm 0.16$ | $0.1 \pm 0.1$ |
| 2 ($\mu$) | $-0.45 \pm 0.25$ | $0.49 \pm 0.28$ | $0.69 \pm 0.18$ |
| 3 ($\tau$) | $0.34 \pm 0.29$ | $-0.60 \pm 0.23$ | $0.67 \pm 0.18$ |

Table 1: Values of the MNS lepton flavour/neutrino mass mixing matrix $U_{ij}$ as derived from solar and atmospheric neutrino oscillation data [2].

Allowing also for the coupling of the state $\nu_3$ to muons, as suggested by data on oscillations of atmospheric neutrinos, but still setting $U_{e3} = 0$ [7] gives, in place of Eqn(8):

$$\left(\frac{\cos\theta_{12} + \sin\theta_{12}}{(\cos\theta_{12} - \sin\theta_{12})\cos\theta_{23} + \sin\theta_{23}}\right)^2 = 0.9976 \pm 0.0032 \qquad (9)$$

Assuming $\sin\theta_{12} = 1/2$, and $\sin\theta_{23} = \cos\theta_{23} = 1/\sqrt{2}$ as suggested in Reference [7] (values consistent with the MNS matrix elements shown in Table 1) gives the value 2 for the LHS of Eqn(9). It is clear, from these considerations, that the hypothesis that a coherent 'lepton flavour eigenstate' is produced in pion decay is experimentally excluded, with an enormous statistical significance, by the experimental measurements of $R_{e/\mu}$ and the MNS matrix elements.

The correct prediction of the pion decay widths, that assumes *incoherent production* of the different neutrino mass eigenstates, is:

$$\begin{aligned}
\Gamma(\pi^- \to \ell^-\nu) &= \Gamma(\pi^- \to \ell^-\nu_1) + \Gamma(\pi^- \to \ell^-\nu_2) + \Gamma(\pi^- \to \ell^-\nu_3) \\
&\simeq \mathcal{M}_{\ell_0}^2(|U_{\ell 1}|^2 + |U_{\ell 2}|^2 + |U_{\ell 3}|^2) = \mathcal{M}_{\ell_0}^2 \qquad (10)
\end{aligned}$$

where the unitarity of the MNS matrix is invoked. The predicted value of $R_{e/\mu}$, given by setting $\theta_{12} = 0$ in Eqn(7), is then independent of the values of the elements of the MNS matrix. Unlike in the case where the coherent production of a 'lepton flavour eigenstate' is assumed, no information about the MNS matrix can be obtained from measurements of $R_{e/\mu}$

Giunti has claimed [8] that the argument just presented is flawed and the coherent states of massive neutrinos are produced in weak decay processes. To substantiate this claim it is suggested to define a 'lepton flavour eigenstate' not according to Eqn(3) but by instead writing the pion decay amplitude as:

$$\mathcal{M}_{\ell\ell}^G \equiv \mathcal{M}_{\ell 1}U_{1\ell} + \mathcal{M}_{\ell 2}U_{2\ell} + \mathcal{M}_{\ell 3}U_{3\ell} = \frac{G}{\sqrt{2}} f_\pi m_\pi V_{ud}\overline{\psi}_\ell(1-\gamma_5)\psi_{\overline{\nu}_\ell}^G \qquad (11)$$

where

$$\begin{aligned}
\psi_{\overline{\nu}_\ell}^G &\equiv U_{\ell 1}U_{1\ell}\psi_{\overline{\nu}_1} + U_{\ell 2}U_{2\ell}\psi_{\overline{\nu}_2} + U_{\ell 3}U_{3\ell}\psi_{\overline{\nu}_3} \\
&= |U_{\ell 1}|^2\psi_{\overline{\nu}_1} + |U_{\ell 2}|^2\psi_{\overline{\nu}_2} + |U_{\ell 3}|^2\psi_{\overline{\nu}_3} \\
&\simeq (|U_{\ell 1}|^2 + |U_{\ell 2}|^2 + |U_{\ell 3}|^2)\psi_{\overline{\nu}_0} = \psi_{\overline{\nu}_0} \qquad (12)
\end{aligned}$$

where $\psi_{\overline{\nu}_0}$ denotes the wavefunction of a massless anti-neutrino, the unitarity of the MNS matrix: $|U_{\ell 1}|^2 + |U_{\ell 2}|^2 + |U_{\ell 3}|^2 = 1$, has been used, and, as in Eqn(5), the purely kinematical effects of non-vanishing anti-neutrino masses are neglected. Since the MNS elements

do not appear in Eqn(11), the prediction given by this equation for $R_{e/\mu}$ is the same as the text-book massless anti-neutrino result, which is in excellent agreement with experiment and provides no information on the values of the MNS elements. However, since the amplitude (11) has no dependence on the values of the MNS elements, so that, unlike the correct Standard Model amplitude (2), the neutrino mass eigenstates are absent, it does not predict neutrino oscillations following pion decay. This seems now to be experimentally excluded by the observation of such oscillations in both atmospheric neutrinos [9] and the K2K [10] experiment. Actually, the anstaz of Eqn(11) which seems to have been constructed precisely to avoid the constraint provided by Eqn(7), is in contradiction with the correct Standard Model expression for the pion decay amplitude,(2), which is linear, not quadratic, in the MNS elements, and does contain the wavefunction of the mass eigenstate $\overline{\nu}_i$. As shown below, neutrino oscillations are predicted by the amplitude (2), if eigenstates with non-degenerate masses exist.

Although the incoherent nature of the production of the different neutrino mass eigenstates, as exemplified in Eqn(10) above, was pointed out more than 20 years ago by Shrock [11, 12], and the unphysical nature of coherent states of neutrinos of different mass was also discussed in the literature [13] the production of a coherent 'lepton flavour eigenstate' at a fixed time remains the basic assumption, in the literature, for the calculation of the phase of neutrino oscillations [7]. The phase of each mass eigenstate is then assumed to evolve with time according to the Schrödinger Equation in its own rest frame (the so-called 'plane-wave' approximation). The assumption that all mass eigenstates are produced at the same time implicitly assumes equal velocities, since there is evidently a unique detection event at some well defined point in space-time. Still, in the derivation of the phase, the neutrino velocities, as defined by the kinematical relation: $v = p/E$ are assumed to be different. Thus contradictory hypotheses are made in space-time and in momentum space. These assumptions lead to the so-called 'standard formula' for the oscillation phase [7]:

$$\phi_{ij}^{stand} = \frac{(m_i^2 - m_j^2)}{2p_\nu} L + O(m^4) \tag{13}$$

where $m_i$ $(i = 1, 2, 3)$ are the neutrino masses, $L$ is the distance between the source pion and the detection event and $p_\nu$ is the neutrino momentum in pion decay to a massless neutrino. Units with $\hbar = c = 1$ are used.

It has been shown recently [14] that the factor of two in the denominator of this equation is a consequence of a common production time for all mass eigenstates, that follows from the incorrect assumption that a coherent 'lepton flavour eigenstate' is produced. As will be demonstrated below, allowing for the possibility of different production times, due to the incoherent nature of the neutrino production process, doubles the contribution of neutrino propagation to the oscillation phase.

Other kinematical approximations that have been made in the literature, e.g. equal momenta and different energies or equal energies and different momenta, result in changes of only O($m^4$) to the phase, as compared to the exact result obtained by imposing both energy and momentum conservation in the neutrino production process. Thus all these results are equivalent at O($m^2$).

In several previous papers written by the present author [14, 15, 16] results for the oscillation phase differing significantly from Eqn(11) have been obtained. These calculations were based on the covariant Feynman Path Integral formulation of quantum mechanics.

A general discussion of the foundations and applications of this approach is given in [17]. Here a concise re-derivation of the probability for $e^-$ production resulting from the interaction of neutrinos from the decay $\pi^+ \rightarrow \mu^+\nu$ at rest will be given. For simplicity, the effect of the finite pion lifetime as well as that of the smearing of the physical masses of the $\pi$ and $\mu$ is neglected. As shown in Reference [15], the corresponding corrections to the oscillation formula are completely negligible. The basis of the calculation is very simple. Each interfering path amplitude has a common initial state (a pion that comes to rest in a stopping target at laboratory time $t = t_0$) and a common final state (defined by the neutrino detection event). There is one such path amplitude for each neutrino mass eigenstate. As neither the neutrino's flavour nor its production time is observed, the corresponding amplitudes must be coherently summed. Quantum mechanical coherence is a property of the path amplitudes (because they all have the same initial and final state) not of the neutrino production process. Indeed, the calculation is a direct application of the fundamental Eqn(7) of Feynman's original Path Integral paper [18].

As in the usual 'plane-wave' derivation of Eqn(13) [7], the space-time propagators of the neutrino and the source pion can be simply obtained by solving the time-dependent Schrödinger Equation[a] in the particle rest frame. The path amplitudes for the detection of an $e^-$ produced in the quasi elastic scattering process: $\nu n \rightarrow e^- p$ by a neutrino from the decay at rest: $\pi^+ \rightarrow \mu^+\nu$ are then:

$$\begin{aligned} A_{e\mu}(i) &= A_0 U_{ei} \exp\{-im_i[\tau_D(\nu) - \tau_i(\nu)]\} U_{\mu i} \exp\{-im_\pi[\tau_i(\pi) - \tau_0(\pi)]\} \\ &= A_0 U_{ei} U_{\mu i} \exp(-i\Delta\phi_i) \quad (i = 1,2,3) \end{aligned} \tag{14}$$

where:

$$A_0 \equiv \langle e^- p|T|\nu n\rangle\langle \nu\mu^+|T|\pi^+\rangle \tag{15}$$

The 'reduced' transition amplitudes $\langle e^- p|T|\nu n\rangle$, $\langle \nu\mu^+|T|\pi^+\rangle$ are defined by factorising out the MNS matrix elements, e.g.:

$$\langle \nu_i\mu^+|T|\pi^+\rangle = U_{\mu i}\langle \nu\mu^+|T|\pi^+\rangle \tag{16}$$

In Eqn(14), $\tau_0(\pi)$, $\tau_i(\pi)$, $\tau_i(\nu)$ and $\tau_D(\nu)$ are the respective proper times of the $\pi$ or $\nu$ at which the pion comes to rest in the stopping target, the neutrino $\nu_i$ is produced and the neutrino is detected. The overall propagator phase of the amplitude $A_{e\mu}(i)$ is the Lorentz invariant quantity:

$$\Delta\phi_i = m_i[\tau_D(\nu) - \tau_i(\nu)] + m_\pi[\tau_i(\pi) - \tau_0(\pi)] \tag{17}$$

Since the pion is at rest:

$$\tau_i(\pi) - \tau_0(\pi) = t_i - t_0 = t_D - t_0 - \frac{L}{v_i} = t_D - t_0 - L\left[1 + \frac{m_i^2}{2p_\nu^2}\right] + O(m^4) \tag{18}$$

where $v_i$ is the velocity of $\nu_i$. The exact kinematical relation for the neutrinos:

$$\Delta\tau = \frac{\Delta t}{\gamma} = \frac{mL}{Ev} = \frac{mL}{p} \tag{19}$$

[a] Note that this equation differs from that conventionally used in atomic physics by the inclusion of the rest-mass energy of the particle. Since the latter is at rest, the kinetic energy vanishes, and the equation is relativistically exact: $\sqrt{m^2 + p^2} = m + T = m$ when $p = T = 0$.

and Eqn(18), substituted in Eqn(17), then give:

$$\Delta\phi_i = \frac{m_i^2}{p_\nu}\left[1-\frac{m_\pi}{2p_\nu}\right]L + m_\pi(t_D - t_0 - L) + O(m^4) \tag{20}$$

Eqns(14) and (20) then give, for the probability of $e^-$ detection distance $L$ from the source:

$$\begin{aligned} P_{e\mu}(L) &= |A_{e\mu}(1)+A_{e\mu}(2)+A_{e\mu}(3)|^2 \\ &= 4|A_0|^2\left[U_{e1}^*U_{\mu1}^*U_{e2}U_{\mu2}\sin^2\frac{\phi_{12}}{2} + U_{e2}^*U_{\mu2}^*U_{e3}U_{\mu3}\sin^2\frac{\phi_{23}}{2}\right. \\ &\quad \left. +U_{e3}^*U_{\mu3}^*U_{e1}U_{\mu1}\sin^2\frac{\phi_{31}}{2}\right] \end{aligned} \tag{21}$$

where

$$\phi_{ij} = \frac{\Delta m_{ij}^2}{p_\nu}\left[1-\frac{m_\pi}{2p_\nu}\right]L + O(m^4) \tag{22}$$

and

$$\Delta m_{ij}^2 = m_i^2 - m_j^2 \tag{23}$$

For neutrinos produced in the decay of an arbitary particle at rest [14, 15], or the decay, in flight, of an arbitary ultra-relativistic particle [15], Eqn(22) generalises to:

$$\phi_{ij} = \frac{\Delta m_{ij}^2 L}{p_\nu}\frac{R_m^2}{(1-R_m^2)} + O(m^4) \tag{24}$$

where $R_m = m_X/m_S$ and $m_X$ is the mass of the particle (or system of particles) recoiling against the neutrino produced in the decay of a source particle of mass $m_S$. The neutrino is assumed to be light so that $m_S \gg m_i$. For $\beta$-decay at rest, the oscillation phase is given by [14, 15]:

$$\phi_{ij} = \frac{\Delta m_{ij}^2}{p_\nu}\left[1-\frac{E_\beta}{2p_\nu}\right]L + O(m^4) \tag{25}$$

where $E_\beta$ is the total energy release of the $\beta$-transition.

The first term in the square brackets in Eqns(22) and (25) is the contribution to the oscillation phase of the neutrino propagator. It is factor of two larger than the similar contribution given by the standard formula (11). The second term in the square brackets in Eqns(22) and (25) is the contribution to the oscillation phase from the propagator of the source particle. It is opposite in sign to the neutrino contribution. It can be seen from Eqn(24), that, for light recoil masses such that $R_m \ll 1$, the oscillation phase tends to vanish, strong cancellation occuring between the neutrino and source particle contributions.

Note that the phase of the propagator of the source particle is the same, at the same time, in the different path amplitudes. The source may therefore be described as 'coherent'. Since, however, the decay process amplitudes are incoherent (see Eqn(10) above) the decay may occur at different times in different path amplitudes. It is this time difference that is at the origin of the contribution of the source particle propagator to the oscillation phase. In physical optics, the coherent source actually gives the entire interference phase, the contributions of photon propagators vanishing [17].

Examination of Eqn(21) shows that the mechanism that governs the value of $P_{e\mu}$ is interference between the path amplitudes for different neutrino flavours. A small value of $P_{e\mu}$ is not necessarily an indication of an approximate conservation of lepton flavour, but may be due to strong destructive interference between the different path amplitudes, independently of the values of the MNS matrix elements. This is made clear by consideration of the two flavour mixing scenario where $U_{e3} = 0$ and Eqn(21) simplifies, using Eqn(6), to:

$$P_{e\mu}(L) = |A_0|^2 \sin^2 2\theta_{12} \sin^2 \frac{\phi_{12}}{2} = \frac{1}{2}|A_0|^2 \sin^2 2\theta_{12}(1 - \cos\phi_{12}) \qquad (26)$$

The term $-\cos\phi_{12}$ in Eqn(26) originates in the interference of the path amplitudes corresponding to $\nu_1$ and $\nu_2$. For small values of $L$, $e^-$ production is suppressed by the almost complete destructive interference of these amplitudes, independently of the value of $\theta_{12}$ i.e. of the degree of non-conservation of lepton number. The destructive nature of the interference is due to the minus sign multiplying $\sin\theta_{12}$ in the second row of the matrix on the RHS of Eqn(6). This, in turn, is a consequence of the unitarity of the MNS matrix.

Indeed, nowhere in the description of the '$\nu_e$ appearence' experiment, just presented, do 'lepton flavour eigenstates' occur, although such an experiment is typically referred to [7] as '$\nu_\mu \to \nu_e$ flavour oscillation'. In fact, only the mass eigenstates $\nu_1$, $\nu_1$ and $\nu_3$ appear in the amplitudes of the physical processes which interfere. It is the interference of these amplitudes in the production of the detection event that constitutes the phenomenon of 'neutrino oscillations'. Indeed, no temporal oscillations of 'lepton flavour' actually occur. Still the terms '$\nu_e$', '$\nu_\mu$' and '$\nu_\tau$' may still have a certain utility as mnemonics, even though they do not represent physical neutrino states for massive neutrinos. For example, it makes sense to refer to solar neutrinos, in a loose way, as a '$\nu_e$ beam ' since the different physical components are all created together with an electron. Similarly, atmospheric neutrinos are predominantly '$\nu_\mu$', i.e., born together with a muon.

The values of the correction factor $C$ that must be applied to a value of $\Delta m^2_{ij}$ obtained using the standard formula (11), to obtain the corresponding quantity as derived from the path amplitude formulae of Eqns(22), (24) or (25) are presented in Table 2 for different neutrino sources. The correction factor is:

$$C = \frac{\phi_{ij}^{stand}}{\phi_{ij}} = \frac{(\Delta m^2_{ij})^{stand}}{\Delta m^2_{ij}} = 1/(\frac{E_S}{p_\nu} - 2) = \frac{1 - R_m^2}{2R_m^2} \qquad (27)$$

where $E_S = m_S$ for particle decays and $E_S = E_\beta$ for $\beta$-decays.

For muon decays and purely leptonic $\tau$ decays, at the mean decay neutrino energy, as in the first row of Table 2, the correction to $(\Delta m^2_{ij})^{stand}$ is only 16 %. The small recoil masses in the decays: $\pi \to e\nu$, K $\to e\nu$ lead to very large values of $C$ (i.e. small values of the oscillation phase). For the electron capture process in the last row of Table 2, which is important for solar neutrino production, the correction factor is unity. The same is true in $\beta$-decay processes when $p_\nu/p_\nu^{max} = 2/3$, close to the mean value $\langle p_\nu \rangle = (11/16)p_\nu^{max}$ of the spectrum of allowed $\beta$-decays. There is then hope that mass differences derived from reactor neutrino experiments [19] using the formula (13) will not be too different from those given by the path amplitude formula (25). Actually, the phase should be evaluated separately for each $\beta$-decay process contributing to the reactor flux, and a suitable weighted average performed. This is evidently a complex and difficult

| Process | Kinematical Condition | $C$ |
|---|---|---|
| $\mu \to e\nu\nu$<br>$\tau \to e\nu\nu$<br>$\tau \to \mu\nu\nu$ | $\langle p_\nu \rangle = 7m_\ell/20$ | 1.16 |
| $\pi \to \mu\nu$ | - | 0.372 |
| $\pi \to e\nu$ | - | 37300 |
| $K \to \mu\nu$ | - | 10.4 |
| $K \to e\nu$ | - | 466718 |
| $K \to \pi\mu\nu$ | $m_X = m_\pi + m_\mu$ ($C^{max}$) | 1.52 |
| $K \to \pi e\nu$ | $m_X = m_\pi + m_e$ ($C^{max}$) | 5.71 |
| ${}^Z N_A \to {}^{Z-1} N_A + e^+ + \nu$ | $p_\nu/p_\nu^{max} = 2/3$<br>e.g. $E_\nu = 9.3$ MeV<br>for ${}^8B \to {}^8 Be^* + e^+ + \nu$ | 1.0 |
| $e^- + {}^Z N_A \to {}^{Z-1} N_A + \nu$ | e.g. $e^- + {}^7 Be \to {}^7 Li + \nu$ | 1.0 |

Table 2: Values of the correction factor $C$ relating the oscillation phase, as calculated using the standard formula, to that calculated using path amplitudes for various neutrino sources.

undertaking. In contrast the corrections for atmospheric neutrinos (from $\pi \to \mu\nu$ and $\mu \to e\nu\nu$ decays) and high energy solar neutrinos, essentially uniquely from ${}^8B$ $\beta$-decay (as shown in the last-but-one row of Table 2), can be applied in a straightforward manner.

Direct experimental discrimination between the standard formula for the oscillation phase (13) and the path amplitude formula (24) is possible in long-baseline terrestrial oscillation experiments by comparing '$\nu_\mu$ disappearence' as observed in neutrino beams produced by either $\pi \to \mu\nu$ or K $\to \mu\nu$ decays. Because of the relatively smaller recoil mass in the second process there is a strong relative suppression of the interference phase. For fixed neutrino momentum and source-detector distance, and considering only '$\nu_\mu \to \nu_\tau$ oscillations':

$$\phi_{23}(\mathrm{K}) = \frac{C_\pi}{C_\mathrm{K}}\phi_{23}(\pi) = 0.0358\phi_{23}(\pi) \tag{28}$$

The K2K experiment with a 250 km baseline has recently found indications of '$\nu_\mu$ disappearence' for neutrinos of average energy 1.3 GeV from the process $\pi \to \mu\nu$ [10]. Using the standard formula (13) the event best fit to the data gives $\Delta m^2_{23} = 2.8 \times 10^{-3}$ eV$^2$ and $\theta_{23} \simeq \pi/4$ [10]. With $L = 250$km, Eqn(13) gives a phase $\phi_{23} = 1.37$ rad. Inserting these values into the standard two flavour '$\nu_\mu$ disappearence' formula [7]:

$$P_{\mu\mu}(L) \simeq 1 - \sin^2 2\theta_{23} \sin^2 \frac{\phi_{23}}{2} \tag{29}$$

gives a suppression factor of 0.6 as compared to the no-oscillation ($\phi_{23} = 0$) case. The standard formula (13) predicts exactly the same phase and suppression factor for neutrinos of the same average momentum produced in K $\to \mu\nu$ decays. However, if Eqns(24) and (28) are correct, $\phi_{23}(K) = 49$ mrad and a reduction of only 0.06 % of the '$\nu_\mu$ flux' is predicted. This crucial test of the basic quantum mechanics of neutrino oscillations requires the installation of separated low energy charged kaon beams in the JHF-SK complex [20], or in similar long-linebase neutrino facilities.

Another possible test is to search for an energy dependence of the oscillation phase for atmospheric neutrinos. Due to the thickness of the Earth's atomsphere, primary cosmic ray interactions that produce the pion ('$\nu_\mu$') and muon ('$\nu_\mu$' and '$\nu_e$') neutrino source particles occur at high altitude ($\simeq$ 20km).For low neutrino energies, the flux of '$\nu_\mu$' is roughly double that of '$\nu_e$' since most muons decay before striking the surface of the Earth. However, for high neutrino energies ($\geq$ 10GeV) most muons cross the atmosphere without decaying due to the relativistic time dilatation effect, so that only pion decays contribute appreciably to the neutrino flux which is thus predominantly ('$\nu_\mu$'. The oscillation phase for in-flight decays, $\phi_{ij}^{IF}$, has been derived, using the Feynman path amplitude approach, in [15] with the result:

$$\phi_{ij}^{IF} = \frac{R_m^2}{(1-R_m^2)}\frac{\Delta m_{ij}^2}{\cos\theta_\nu E_\nu}L = \left(\frac{1}{2C}\right)\frac{\Delta m_{ij}^2}{\cos\theta_\nu E_\nu}L \tag{30}$$

where $\theta_\nu$ is the angle between the directions of the neutrino and its source particle in the laboratory and $E_\nu$ is the laboratory neutrino energy. For $\pi \rightarrow \mu\nu$ decays the factor $(1/2C)$ has the value 1.34, whereas for $\mu \rightarrow e\nu\overline{\nu}$ decays the value of $C$ corresponding to $\langle p_\nu \rangle$ quoted in Table 2 gives the value $(1/2C) = 0.431$. Thus the oscillation phase is predicted by the path amplitude calculation to be larger at high values of $E_\nu$ where pion decays are the predominant neutrino source. This prediction may be tested using neutrino interaction data from the IceCube neutrino detector [21]. In a ten year exposure to atmospheric neutrinos more than $7 \times 10^5$ events are expected with muon energies $E_\mu$ greater than 100 GeV [22]. For such events neutrino oscillations are strongly suppressed due to the $(p_\nu)^{-1}$ factor in oscillation phase formulae. However, for lower energies, 10 GeV $<$ $E_\mu$ $<$ 100 GeV, where much larger numbers of events are expected, oscillations can be observed and the detected neutrinos are expected to originate almost entirely from pion decay. Thus the prediction of the path amplitude formula can be tested by comparing values of $\Delta m_{23}^2$ obtained at low energies from water Cerenkov detectors such as SuperKamiokande, where pion and muon decays contribute almost equally, with future IceCube results dominated by pion decay neutrinos.

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# References


[1] Z.Maki, M.Nakagawa and S.Sakata, Prog. Theor. Phys. **28** 870 (1962).

[2] M.C.Gonzalez-Garcia and Y.Nir, Rev. Mod. Phys. **75** 345 (2003).

[3] N.Cabibbo, Phys. Rev. Lett. **10** 531 (1963);
M.Kobayashi and T.Maskawa, Prog. Theor. Phys. **49** 652 (1973).

[4] W.Marciano and A.Sirlin, Phys. Rev. Lett. **36** 1425 (1976).

[5] T.Goldman and W.Wilson, Phys. Rev. **D15** 709 (1977).

[6] *Review of Particle Properties*, H.Hagiwara *et al.*, Phys. Rev. **D66** 010001 (2002).

[7] B.Kayser, *Neutrino Physics as Explored by Flavour Change* in Phys. Rev. **D66** 010001 (2002) (Review of Particle Properties), and References therein.

[8] C.Giunti, *Coherence in Neutrino Oscillations*, http://xxx.lanl.gov/abs/hep-ph/0302045.

[9] T.Kajita and Y.Totsuka, Rev. Mod. Phys. **73** 85 (2001).

[10] M.H.Ahn *et al.*, Phys. Rev. Lett. **90** 041801 (2003).

[11] R.E.Shrock, Phys. Lett. **B96** 159 (1980).

[12] R.E.Shrock, Phys. Rev. **D24** 1232 (1981); **D24** 1275 (1981).

[13] C.Giunti, C.W.Kim and U.W.Lee, Phys. Rev. **D45** 2414 (1992).

[14] J.H.Field, Eur. Phys. J. **C30** 305 (2003).

[15] J.H.Field, Eur. Phys. J. **C37** 359 (2004).

[16] J.H.Field, *Quantum Interference Effects in the Detection Probability of Charged Leptons Produced in Charged Current Weak Interactions*, http://xxx.lanl.gov/abs/hep-ph/0303152.

[17] J.H.Field, Ann. Phys. (NY) **321** 627 (2006).

[18] R.P.Feynman, Rev. Mod. Phys. **20** 367 (1948).

[19] K.Eguchi *et al.*, Phys. Rev. Lett. **90** 021802 (2003).

[20] Y.Itow *et al.*, *The JHF to Kamioka Project*, KEK report 2001-4, 2001, http://xxx.lanl.gov/abs/hep-ex/0106019.

[21] http://icecube.wisc.edu/

[22] M.C.Gonzalez-Garcia, F.Halzen and M.Maltoni, Phys.Rev. **D71** 093010 (2005).